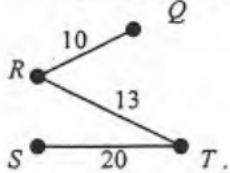
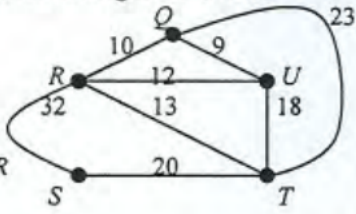


<p>1</p> <p>Original list    6 5 9 4 5 2</p> <p>After 1<sup>st</sup> pass    <span style="border: 1px solid black; padding: 2px;">5 6</span> 9 4 5 2</p> <p>After 2<sup>nd</sup> pass    <span style="border: 1px solid black; padding: 2px;">5 6 9</span> 4 5 2</p> <p>After 3<sup>rd</sup> pass    <span style="border: 1px solid black; padding: 2px;">4 5 6 9</span> 5 2</p> <p>After 4<sup>th</sup> pass    <span style="border: 1px solid black; padding: 2px;">4 5 5 6 9</span> 2</p> <p>After 5<sup>th</sup> pass    2 4 5 5 6 9</p> <p>May be shown vertically</p>	<p>M1</p> <p>M1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p style="text-align: right;"><span style="border: 1px solid black; padding: 2px;">5</span></p>	<p>Decreasing order can score method n</p> <p>For 1<sup>st</sup> pass correct with shuttle sort</p> <p>For 2<sup>nd</sup> pass correct with shuttle sort or follow through from previous list</p> <p>For 3<sup>rd</sup> pass correct with shuttle sort or follow through from previous list</p> <p>For 4<sup>th</sup> pass correct with shuttle sort or follow through from previous list</p> <p>For final list from correct shuttle sort, with results at end of each pass clearly shown</p>
<p>2</p> <p>(i) Number of arcs <math>\times 2 =</math> sum of orders of vertices  <math>\Rightarrow (3+3+4+4+4+4) \div 2 = 11</math> arcs</p> <p>-----</p> <p>(ii) Semi-Eulerian, it has exactly two odd vertices</p> <p>-----</p> <p>(iii) Complete graph on five vertices has only 10 arcs, so 11 arcs means that all six vertices are connected.</p> <p>Or, a vertex of order 4 must join to four others so five vertices are connected. The sixth vertex has order at least three and cannot connect to itself so it must join to the other five.</p> <p>Or any equivalent reasoning.</p>	<p>M1</p> <p>A1</p> <p>B1</p> <p>B2</p> <p style="text-align: right;"><span style="border: 1px solid black; padding: 2px;">5</span></p>	<p>For a general method</p> <p>For 11 calculated</p> <p>Drawing a specific case to get 11 scores B1 only</p> <p>For semi-Eulerian with a valid reason</p> <p>Accept 'two odd nodes' or 'two nodes of order 3' as minimal reasons</p> <p>For a good explanation of the general case by considering orders of vertices</p> <p>A weak explanation may score B1</p> <p>A diagram of a specific case is not sufficient</p>
<p>3</p> <p>(i) Minimum spanning tree with <math>U</math> removed</p>  <p><math>QR + RT + TS = 43</math> miles</p> <p>Join <math>U</math> back in using two shortest arcs</p> <p><math>43 + 9 + 12</math></p> <p><math>= 64</math> miles</p> <p>-----</p> <p>(ii) Trying to apply nearest neighbour method</p>  <p>Start from <math>R</math></p> <p>to give <math>RQUTSR</math></p> <p><math>= 89</math> miles</p>	<p>M1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>A1</p> <p style="text-align: right;"><span style="border: 1px solid black; padding: 2px;">7</span></p>	<p>For 43 or arcs <math>QR, RT, TS</math> or a convincing attempt to find minimum spanning tree for <math>\{Q, R, S, T\}</math></p> <p>For their <math>43 + 9 + 12</math></p> <p>cao (miles may be implied)</p> <p>For a correct start to an application of nearest neighbour with any start vertex, ie at least:  <math>QURTS, STRQU, TRQU</math> or <math>UQRTS</math></p> <p>For <math>R</math> as start vertex (may be implied from cycle)</p> <p>For <math>RQUTSR</math></p> <p>For 89 (miles may be implied) from valid method</p>

4	(i)	<table border="1"> <thead> <tr> <th></th> <th>1<sup>st</sup></th> <th>3<sup>rd</sup></th> <th>2<sup>nd</sup></th> <th>4<sup>th</sup></th> <th>5<sup>th</sup></th> <th>7<sup>th</sup></th> <th>6<sup>th</sup></th> <th>8<sup>th</sup></th> </tr> <tr> <th></th> <th>A</th> <th>B</th> <th>C</th> <th>D</th> <th>E</th> <th>F</th> <th>G</th> <th>H</th> </tr> </thead> <tbody> <tr> <th>A</th> <td>-</td> <td>4</td> <td>2</td> <td>3</td> <td>-</td> <td>-</td> <td>-</td> <td>-</td> </tr> <tr> <th>B</th> <td>4</td> <td>-</td> <td>1</td> <td>-</td> <td>3</td> <td>-</td> <td>-</td> <td>-</td> </tr> <tr> <th>C</th> <td>2</td> <td>1</td> <td>-</td> <td>2</td> <td>-</td> <td>6</td> <td>5</td> <td>-</td> </tr> <tr> <th>D</th> <td>3</td> <td>-</td> <td>2</td> <td>-</td> <td>-</td> <td>-</td> <td>4</td> <td>-</td> </tr> <tr> <th>E</th> <td>-</td> <td>3</td> <td>-</td> <td>-</td> <td>-</td> <td>8</td> <td>-</td> <td>7</td> </tr> <tr> <th>F</th> <td>-</td> <td>-</td> <td>6</td> <td>-</td> <td>8</td> <td>-</td> <td>-</td> <td>8</td> </tr> <tr> <th>G</th> <td>-</td> <td>-</td> <td>5</td> <td>4</td> <td>-</td> <td>-</td> <td>-</td> <td>9</td> </tr> <tr> <th>H</th> <td>-</td> <td>-</td> <td>-</td> <td>-</td> <td>7</td> <td>8</td> <td>9</td> <td>-</td> </tr> </tbody> </table>		1 <sup>st</sup>	3 <sup>rd</sup>	2 <sup>nd</sup>	4 <sup>th</sup>	5 <sup>th</sup>	7 <sup>th</sup>	6 <sup>th</sup>	8 <sup>th</sup>		A	B	C	D	E	F	G	H	A	-	4	2	3	-	-	-	-	B	4	-	1	-	3	-	-	-	C	2	1	-	2	-	6	5	-	D	3	-	2	-	-	-	4	-	E	-	3	-	-	-	8	-	7	F	-	-	6	-	8	-	-	8	G	-	-	5	4	-	-	-	9	H	-	-	-	-	7	8	9	-	M1 M1 A1 B1	Answer should be on insert For starting by choosing row C in column A For choosing more than one entry from column C For a correct order (A), C, B, D, E, G, F, H For correct entries chosen or a correct tree drawn
			1 <sup>st</sup>	3 <sup>rd</sup>	2 <sup>nd</sup>	4 <sup>th</sup>	5 <sup>th</sup>	7 <sup>th</sup>	6 <sup>th</sup>	8 <sup>th</sup>																																																																																				
			A	B	C	D	E	F	G	H																																																																																				
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		C	2	1	-	2	-	6	5	-																																																																																				
		D	3	-	2	-	-	-	4	-																																																																																				
		E	-	3	-	-	-	8	-	7																																																																																				
		F	-	-	6	-	8	-	-	8																																																																																				
		G	-	-	5	4	-	-	-	9																																																																																				
H	-	-	-	-	7	8	9	-																																																																																						
		Quickest time is at least 25 hours	B1	For 25 Accept 'more than 25'																																																																																										
	(ii)		M1 A1	For a correct graph drawn For correct weights shown																																																																																										
	(iii)	If AC is used then either B or D is excluded. Or must pass through C in getting between B and D, so AC is impossible.	B1	Follow through graph, if possible, provided same conclusion is valid For explaining what happens if AC is used or why AC cannot be included.																																																																																										
	(iv)	If EF is not used then passing through either E or F will take the team to H, the team will not be able to visit both E and F.	B1	Follow through graph, if possible, provided same conclusion is valid For stating the effect of not using arc EF or for considering all possible routes into H																																																																																										
	(v)	<p>ABEFCDGH</p> <p>ADGCBEFH</p> <p>The second route is quicker (32 hours compared with 36 hours)</p>	M1 M1 A1	Follow through graph, if possible For this route For this route For identifying ADGCBEFH as the quicker or for calculating 32																																																																																										

<p>5 (i) <math>x \geq 0, y \geq 0</math> <math>y \leq 2x + 1</math> <math>4x + 3y \leq 12</math></p>	<p>B1 B1 B1</p>	<p>For both trivial constraints; and For this inequality, or equivalent; For this inequality, or equivalent; and</p>																																																								
<p>(ii) (0, 0), (3, 0), (0, 1) (0.9, 2.8) (0, 0) <math>\rightarrow P = 0</math>; (0, 1) <math>\rightarrow P = 3</math>; (0.9, 2.8) <math>\rightarrow P = 12.9</math>; (3, 0) <math>\rightarrow P = 15</math> <math>x = 3</math> and <math>y = 0</math> <math>P = 15</math></p>	<p>B2 B1 M1  A1 A1</p>	<p>For these three vertices, any two correct For this vertex exact, in decimals or fractions For calculating <math>P = 5x + 3y</math> for at least one of their vertices or clear evidence of using an appropriate line of constant profit For the correct values of <math>x</math> and <math>y</math> clearly identified For 15 clearly identified as the optimum value</p>																																																								
<p>(iii) Either consider the gradient of the profit line (<math>-\frac{1}{2}a</math>) and the gradients of the boundary lines (2 and <math>-1\frac{1}{2}</math>) or calculate <math>Q</math> at vertices <math>\Rightarrow 3, 0.9a+8.4, 3a</math> Hence require <math>a \leq -6</math></p>	<p>M1 M1 M1  A1</p>	<p>One method mark for each appropriate gradient calculated correctly or for each appropriate value of <math>Q</math> calculated correctly  For the correct set of values identified [<math>a = -6</math> or any valid proper subset of the correct answer with no method shown <math>\Rightarrow</math> B1 only]</p>																																																								
13																																																										
<p>6 (i) <math>5x + 3y - 5z + s = 15</math> <math>2x + 6y + 8z + t = 24</math></p>	<p>B1 B1</p>	<p>For both equations correctly stated For <math>\pm(-2 \ 5 \ 1)</math> in objective row</p>																																																								
<p>(ii)</p> <table border="1" data-bbox="207 695 749 816"> <thead> <tr> <th><math>P</math></th> <th><math>x</math></th> <th><math>y</math></th> <th><math>z</math></th> <th><math>s</math></th> <th><math>t</math></th> <th>-</th> </tr> </thead> <tbody> <tr> <td>1</td> <td>-2</td> <td>5</td> <td>1</td> <td>0</td> <td>0</td> <td>0</td> </tr> <tr> <td>0</td> <td>5</td> <td>3</td> <td>-5</td> <td>1</td> <td>0</td> <td>15</td> </tr> <tr> <td>0</td> <td>2</td> <td>6</td> <td>8</td> <td>0</td> <td>1</td> <td>24</td> </tr> </tbody> </table> <p>Pivot on 5 in <math>x</math> column</p> <table border="1" data-bbox="207 846 749 977"> <thead> <tr> <th><math>P</math></th> <th><math>x</math></th> <th><math>y</math></th> <th><math>z</math></th> <th><math>s</math></th> <th><math>t</math></th> <th>-</th> </tr> </thead> <tbody> <tr> <td>1</td> <td>0</td> <td>6.2</td> <td>-1</td> <td>0.4</td> <td>0</td> <td>6</td> </tr> <tr> <td>0</td> <td>1</td> <td>0.6</td> <td>-1</td> <td>0.2</td> <td>0</td> <td>3</td> </tr> <tr> <td>0</td> <td>0</td> <td>4.8</td> <td>10</td> <td>-0.4</td> <td>1</td> <td>18</td> </tr> </tbody> </table>	$P$	$x$	$y$	$z$	$s$	$t$	-	1	-2	5	1	0	0	0	0	5	3	-5	1	0	15	0	2	6	8	0	1	24	$P$	$x$	$y$	$z$	$s$	$t$	-	1	0	6.2	-1	0.4	0	6	0	1	0.6	-1	0.2	0	3	0	0	4.8	10	-0.4	1	18	<p>B1 B1 B1  B1 M1 M1 A1</p>	<p>Follow through from part (i) For 5 3 -5 1 0 15 and 2 6 8 0 1 24 or equivalent in constraint rows  For correct pivot choice for their tableau  For a correct method for their table and their pivot choice For increasing <math>P</math> For correct tableau or equivalent, <math>cao</math></p>
$P$	$x$	$y$	$z$	$s$	$t$	-																																																				
1	-2	5	1	0	0	0																																																				
0	5	3	-5	1	0	15																																																				
0	2	6	8	0	1	24																																																				
$P$	$x$	$y$	$z$	$s$	$t$	-																																																				
1	0	6.2	-1	0.4	0	6																																																				
0	1	0.6	-1	0.2	0	3																																																				
0	0	4.8	10	-0.4	1	18																																																				
<p>(iii)</p> <p>Pivot on 10 in <math>z</math> column</p> <table border="1" data-bbox="207 1058 749 1189"> <thead> <tr> <th><math>P</math></th> <th><math>x</math></th> <th><math>y</math></th> <th><math>z</math></th> <th><math>s</math></th> <th><math>t</math></th> <th>-</th> </tr> </thead> <tbody> <tr> <td>1</td> <td>0</td> <td>6.68</td> <td>0</td> <td>0.36</td> <td>0.1</td> <td>7.8</td> </tr> <tr> <td>0</td> <td>1</td> <td>1.08</td> <td>0</td> <td>0.16</td> <td>0.1</td> <td>4.8</td> </tr> <tr> <td>0</td> <td>0</td> <td>0.48</td> <td>1</td> <td>-0.04</td> <td>0.1</td> <td>1.8</td> </tr> </tbody> </table> <p><math>x = 4.8, y = 0, z = 1.8</math> <math>P = 7.8</math></p>	$P$	$x$	$y$	$z$	$s$	$t$	-	1	0	6.68	0	0.36	0.1	7.8	0	1	1.08	0	0.16	0.1	4.8	0	0	0.48	1	-0.04	0.1	1.8	<p>M1 A1  B1 B1</p>	<p>For correct pivot choice  For correct tableau or equivalent, <math>cao</math>  For all three correct values for their final tableau For correct value for their final tableau</p>																												
$P$	$x$	$y$	$z$	$s$	$t$	-																																																				
1	0	6.68	0	0.36	0.1	7.8																																																				
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<p>(iv) We must now pivot on the 2 in the <math>x</math> column, this gives</p> <table border="1" data-bbox="207 1330 749 1461"> <thead> <tr> <th><math>P</math></th> <th><math>x</math></th> <th><math>y</math></th> <th><math>z</math></th> <th><math>s</math></th> <th><math>t</math></th> <th>-</th> </tr> </thead> <tbody> <tr> <td>1</td> <td>0</td> <td>11</td> <td>9</td> <td>0</td> <td>1</td> <td>24</td> </tr> <tr> <td>0</td> <td>0</td> <td>-12</td> <td>-25</td> <td>1</td> <td>-2.5</td> <td><math>k-60</math></td> </tr> <tr> <td>0</td> <td>1</td> <td>3</td> <td>4</td> <td>0</td> <td>0.5</td> <td>12</td> </tr> </tbody> </table> <p>Hence <math>y = 0</math> Accept 'no change to <math>y</math>'</p>	$P$	$x$	$y$	$z$	$s$	$t$	-	1	0	11	9	0	1	24	0	0	-12	-25	1	-2.5	$k-60$	0	1	3	4	0	0.5	12	<p>M1  A1</p>	<p>For showing what happens to tableau, only need to show enough to be able to deduce answer (eg top row: 0 11 9 0 1 or <math>y</math> column)  For correctly deducing <math>y = 0</math> in general case.  Only using a specific value of <math>k</math> (eg <math>k = 60</math>) with no general argument <math>\Rightarrow</math> M1, A0  Do not imply method mark from statement '<math>y = 0</math>' with no method seen.</p>																												
$P$	$x$	$y$	$z$	$s$	$t$	-																																																				
1	0	11	9	0	1	24																																																				
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13																																																										

7 (a) (i)	<p><b>A</b></p> <table border="1" style="display: inline-table; margin-right: 20px;"> <tr><td>1</td><td>0</td></tr> <tr><td colspan="2"> </td></tr> </table> <p><b>B</b></p> <table border="1" style="display: inline-table; margin-right: 20px;"> <tr><td>2</td><td>2</td></tr> <tr><td colspan="2">2</td></tr> </table> <p><b>C</b></p> <table border="1" style="display: inline-table; margin-right: 20px;"> <tr><td>3</td><td>4</td></tr> <tr><td colspan="2">4</td></tr> </table> <p><b>D</b></p> <table border="1" style="display: inline-table; margin-right: 20px;"> <tr><td>4</td><td>7</td></tr> <tr><td colspan="2">8 7</td></tr> </table> <p><b>E</b></p> <table border="1" style="display: inline-table; margin-right: 20px;"> <tr><td>6</td><td>9</td></tr> <tr><td colspan="2">12 11 9</td></tr> </table> <p><b>F</b></p> <table border="1" style="display: inline-table; margin-right: 20px;"> <tr><td>7</td><td>14</td></tr> <tr><td colspan="2">15 14</td></tr> </table> <p><b>G</b></p> <table border="1" style="display: inline-table; margin-right: 20px;"> <tr><td>5</td><td>8</td></tr> <tr><td colspan="2">8</td></tr> </table> <p><b>H</b></p> <table border="1" style="display: inline-table; margin-right: 20px;"> <tr><td>8</td><td>18</td></tr> <tr><td colspan="2">20 18</td></tr> </table> <p><b>J</b></p> <table border="1" style="display: inline-table;"> <tr><td>9</td><td>38</td></tr> <tr><td colspan="2">39 38</td></tr> </table>	1	0			2	2	2		3	4	4		4	7	8 7		6	9	12 11 9		7	14	15 14		5	8	8		8	18	20 18		9	38	39 38		<p>M1 For correct temporary labels at <i>D</i> and <i>E</i> (condone extras here)</p> <p>A1 For all temporary labels correct (with no extras)</p> <p>M1 For value 38 at <i>J</i></p> <p>A1 For all permanent labels correct</p> <p>B1 For the correct order of assigning permanent labels: <i>A, B, C, D, G, E, F, H, J</i></p>
1	0																																					
2	2																																					
2																																						
3	4																																					
4																																						
4	7																																					
8 7																																						
6	9																																					
12 11 9																																						
7	14																																					
15 14																																						
5	8																																					
8																																						
8	18																																					
20 18																																						
9	38																																					
39 38																																						
	<p>Shortest route: <i>A - B - G - E</i> Length = 900</p> <p>Shortest route: <i>A - C - D - F - H - J</i> Length = 3800</p>	<p>B1 For correct route and length. Accept route reversed and accept length = 9</p> <p>B1 For correct route and length. Accept route reversed and accept length = 38</p>																																				
(ii)	<p>Length: 4700 metres</p> <p><i>E - G - B - A - C - D - F - H - J</i></p>	<p>Follow through from (i), if possible</p> <p>B1 For 47 or 4700</p> <p>M1 For <i>E - G - B - A</i>, or reversed, as part of a longer route</p> <p>M1 For <i>A - C - D - F - H - J</i>, or reversed, as part of a longer route</p> <p>A1 For whole route correct</p>																																				
(iii)	<p>Explanation: <i>G - B - A - C - D - F - H</i> <i>E</i> and <i>J</i> will be left out (either is sufficient)</p>	<p>May be implied</p> <p>M1 For identifying that route will not visit every vertex.</p>																																				
(b)	<p>Odd nodes are <i>A, C, D, E, F, G</i> Need to pair <i>C, D, F, G</i> in the shortest way</p> <p><math>CD = 3</math> and <math>FG = 7 \Rightarrow 10</math> (<math>CF = 10, DG = 11</math> and <math>CG = 8, DF = 7</math>)</p> <p>Sum of all weights = 147</p> <p>Length = <u>15700 metres</u></p>	<p>M1 For trying to pair <i>C, D, F, G</i> (and no others)</p> <p>A1 For <i>CD, FG</i> or 10 (or 1000)</p> <p>M1 For 147 (or 14700) or a good attempt seen or implied</p> <p>A1 For 15700 metres (or 15700 m or 157 hundred metres or 15.7 km). But <math>157 \Rightarrow M1, A0</math></p>																																				